Faculty of Science

MATA31, Mathematics: Analysis in One Variable, 15 credits

Matematik: Envariabelanalys, 15 högskolepoäng

First Cycle / Grundnivå

Details of approval

The syllabus was approved by Study programmes board, Faculty of Science on 2023-12-11 and was last revised on 2024-02-14. The revised syllabus comes into effect 2024-02-14 and is valid from the autumn semester 2024.

General information

The course is a compulsory course for first-cycle studies for a Bachelor of Science degree in mathematics and in physics. The course can also be given as a stand-alone course or as part of a course package.

Language of instruction: Swedish and English
The course is given alternately in Swedish or English, depending on the course instance. The course is given in Swedish within the Swedish taught specialisations of the Science Bachelor's Programme and as a stand-alone course. The course is given in English within the English taught specialisations of the Science Bachelor's Programme.

Main field of study	*Specialisation*
Mathematics	G1N, First cycle, has only upper-secondary level entry requirements

Learning outcomes

The overarching goal of the course is for students to develop understanding of central concepts, results and methods of analysis in one variable, and to apply these methods to solve standard calculus problems for functions in one variable. The course aims for students to develop the ability to communicate mathematics in speech and writing, as well as reading mathematical texts. The course aims additionally to prepare students for further studies in mathematics and natural sciences.

Knowledge and understanding

On completion of the course, the students shall be able to:

- state and outline the relations between the most important concepts and results included in the course, and illustrate these with examples
- explain how the most important concepts and results of the course are related to methods for solving problems in single variable calculus
- explain how standard concepts of calculus are related to convergence and quantitative estimates with given error bounds.

Competence and skills

On completion of the course, the students shall be able to:

- within the framework of the course with confidence be able to handle elementary functions of one variable, including limits, derivatives and integrals of such functions
- demonstrate a good computational proficiency
- use the methods included in the course to solve problems in single variable calculus
- use the methods included in the course to make quantitative approximations within given error bounds
- present solutions to problems in single variable calculus in speech and in writing, logically coherent and with adequate terminology
- derive basic relations between key concepts, and reproduce proofs of the most important results included in the course.

Judgement and approach

On completion of the course, the students shall be able to:

- argue for why proofs are necessary when developing a mathematical theory
- explain how the most important concepts of the course are related to quantitative approximations
- argue for the importance of mathematical analysis in other sciences
- analyse critically other students' solutions and presentations and evaluate alternative solutions in relation to their own solutions.

Course content

The course treats:

- The real numbers: axioms, examples of proofs of basic arithmetical rules.
- The elementary functions, polynomials, rational functions, the exponential function and the natural logarithm, the trigonometric functions and the inverse trigonometric functions; definitions, basic properties, and quantitative approximations using representations in terms of areas and arclengths.
- Sequences of numbers and their limits: formal definition of the limit, examples of proofs of their computational rules, visual representation of convergence of recursive sequences, quantitative approximations.

- Infinite series: applications and proofs of convergence tests, absolute convergence, quantitative approximations using partial sums and tail estimates.
- Functions and their limits: formal definition of the limit, proofs and applications of their computational rules, indeterminate forms and asymptotes.
- Continuity: continuity of elementary functions, the intermediate value theorem and the min-max theorem.
- Derivatives: definition, proofs and applications of computational rules, differentiation formulas for elementary functions, Rolle's lemma, the mean value theorem and L'Hopital's rule.
- Applications of the derivative: optimisation and graph sketching, techniques for establishing identities and inequalities.
- Indefinite integrals: proofs and applications of basic computational rules and integration methods, such as change of variables, partial integration and use of partial fraction decomposition.
- Definite integrals: Darboux integrability of monotone functions and functions with bounded derivative with related error estimates, the fundamental theorem of calculus, applications to arclength, rotational volumes and surfaces, numerical approximations of definite integrals.
- Improper integrals: convergence criteria for improper integrals for positive functions, absolute convergence, comparison to infinite series.
- Differential equations: direction fields, analytic solution methods for separable and linear first order differential equations, solution method for linear higher-order differential equations with constant coefficients, numerical approximations of solutions of initial value problems using Euler's method.
- Taylor expansions: Taylor's formula with Lagrange's formula for the error term, uniqueness theorem for Taylor polynomials, numerical approximations of function values and integrals using Taylor polynomials.

In addition, materials on sets, functions and relations, induction, the binomial theorem, as well as variables, for-loops and if-statements in Python are covered at the beginning of the course.

Course design

The teaching consists of lectures, seminars, exercise classes and mentoring sessions. An essential element of the seminars is training in problem solving and mathematical communication and assumes students' active participation. Several compulsory computer-based tests are given during the course. A compulsory assignment providing students with training in mathematical communication in writing as well as a compulsory problem solving assignment are included in the course.

The first part of the course is taught jointly with *Algebra and Vector Geometry* and *Computational Programming in Python*, where material on sets, induction, the binomial theorem, functions and relations, as well as variables, for-loops and if-statements in Python, are covered.

Assessment

The examination consists of the following parts:

- written assignment in mathematical communication (1 credit)
- oral presentation of problem solving assignment (1 credit)

- computer based tests (1 credit)
- mid-term written examination (4.5 credits)
- final written examination with an optional oral examination (7.5 credits)

The written assignment provides students with training in mathematical communication in writing. The students are to select a proof covered in the first part of the course and present it in writing using Latex.

The problem solving assignment aims to strengthen students' understanding of key concepts and their connections to numerical approximations with error bounds. The solutions are presented at one of the seminar sessions and aim to provide students training in oral mathematical communication.

The optional oral examination is offered only to students who obtained at least 70% of the maximum number of the marks in the final written examination and it is required to obtain the grade Pass with distinction on the whole course.

Students who fail the ordinary written examinations are offered a resit examination during scheduled re-examination period.

The examiner, in consultation with Disability Support Services, may deviate from the regular form of examination in order to provide a permanently disabled student with a form of examination equivalent to that of a student without a disability.

Grades

Grading scale includes the grades: Fail, Pass, Pass with distinction
The grading scale for the assignments, the computer-based tests and the mid-term examination is Fail, Pass.

The written examination, possibly together with the optional oral examination, is graded according to the scale Fail, Pass, Pass with Distinction. To achieve the grade Pass with distinction, it is required to obtain at least 70% of the maximum number of the marks in the written examination and demonstrate satisfactory performance at the oral examination.

For a Pass grade on the whole course, the student must have Pass grades on the written assignment, the oral presentation, the computer-based tests and on both written examinations.

The final grade is determined by the grade on the final written examination including the optional oral examination.

Entry requirements

General requirements and studies equivalent of course Mathematics 4/D from Swedish Upper Secondary School.

Further information

The course may not be included in a degree together with the courses Mathematics 1 alpha (MAT131 or MATA11), Mathematics 1 beta (MAT132 or MATA12), Analysis 1 (MATA14), Analysis in One Variable (MATA21), Mathematics for science students (MATA01, MATA02), Mathematics for Scientists 1 (MATA03).

The course is given at the Centre for Mathematical Sciences, Lund University.