Faculty of Science

MASC12, Mathematical Statistics: Statistical Inference Theory, 7.5 credits

Matematisk statistik: Statistisk inferensteori, 7,5 högskolepoäng

First Cycle / Grundnivå

Details of approval

The syllabus was approved by The Education Board of Faculty of Science on 2025-06-02. The syllabus comes into effect 2025-06-02 and is valid from the spring semester 2026.

General information

The course is an elective course for first-cycle studies for a Bachelor of Science in Mathematics

Language of instruction: English

Main field of study	*Specialisation*
Mathematics	G2F, First cycle, has at least 60 credits in first-cycle course/s as entry requirements
Mathematical Statistics	G2F, First cycle, has at least 60 credits in first-cycle course/s as entry requirements

Learning outcomes

The pupose of the course is to given the students deeper knowledge and understanding of estimation and decision theory as well as giving a background for further studies in mathematical statistics.

Knowledge and understanding

On completion of the course, the students are expected to be able to

- explain why there is no optimal estimator or decision in general,

- describe how, by restricting to families of distribution (exponential families or group families) and to allowed types of estimators (unbiased or equivariant), and by changing pointwise risk to overall risk (Bayesian and maximum), obtain optimal solutions.
- describe how the different optimal solutions (uniform minimum variance unbiased (UMVU), minimum risk equivariant estimators (MRE), Bayesian estimators, and minimax estimators) are related to the assumptions in the previous outcome.
- explain the concept of sufficiency, and how it gives rise to randomized estimators,
- state the limit distribution for the maximum likelihood estimator (mle), and explain how the asymptotic variance relates to the Cramér-Rao bound,
- state the likelihood ratio, Wald and score test statistics, and explain how they can be used for testing.

Competence and skills

On completion of the course, the students are expected to be able to

- use the Rao-Blackwell theorem to derive UMVU (or LMVU) estimators
- derive UMVU estimators in exponential families
- derive MRE estimators for location families of distributions
- derive the Bayes estimators for Gaussian data and for binomial data, using appropriate priors
- derive optimal tests on a given level for distributions with monotone likelihood ratios

Judgement and approach

On completion of the course, the students are expected to be able to reflect on

- the fact that there are is in general not an optimal solution to an inference problem, and on the approaches to accommodate for this a) by restricting to allowed estimator, using a pointwise risk for describing optimality b) by using an overall risk, and Bayesian and minimax approaches,
- the use of randomized tests for deriving optimal tests, and why randomization enters,
- the concept of minimal sufficiency as a way to reduce data without information loss,
- the three approaches to inference (point estimation, confidence intervals and tests) as decisions,
- the relation between choice of a priori distribution, empirical data and the resulting a posteriori distribution,
- the relation between Bayes estimators and minimax estimators.

Course content

The course ttreats:

- Exponential families,sufficient statistics, factorization criteria, Rao-Blackwells theorem, ancilliary statistics, Uniform minimum variance(UMVU) estimators.
- Location-scale families of distributions. Minimum risk equivariant estimators(MRE).
- Bayesian inference. Single-prior Bayes estimators. Conjugate priors. Mimimax estimators. Relations between Bayes estimators, minimax estimators and least favourable distributions.
- Decision theory. Point estimation, confidence intervals and tests as decisions. Neyman-Pearson's lemma. Distributions with monotone likelihood ratios. Uniformly most powerful tests.
- Limit distributions for the maximum likelihood estimator. Profile likelihood and marginal likelihood. The Cramér-Rao bound. Likelihood ratio tests, Wald tests and score tests.

Course design

Teaching consists of lectures and exercises where the exercises to a high degree involves active participation of the students. The students should therefore prepare to be able to participate in the discussions and problem solving. The exercises are not mandatory but highly recommended.

Assessment

The examination consists of a written exam followed by an oral exam. Students who fail the regular exam are offered a re-examination shortly afterwards.

The examiner, in consultation with Disability Support Services, may deviate from the regular form of examination in order to provide a permanently disabled student with a form of examination equivalent to that of a student without a disability.

Grades

Grading scale includes the grades: Fail, Pass, Pass with distinction
For a passing grade on the entire course a passing grade on the written and oral exam. The grade is formed by weighing together the results on the written and the oral part of the examination.

Entry requirements

For admission to the course, general entry requirements and 60 credits of studies in mathematics including knowledge equivalent to the course MASA03 Mathematical statistics: Basic course, 15 ECTS. English course 6/B.

Further information

The courses replaces MASC02, Statistical Inference Theory, 7.5 hp and credits from that course cannot count towards a degree together with this course.

The course is given by Centre for Mathematical Sciences, Lund University.